<u>แบบทดสอบพื้นฐานวิชาคณิตศาสตร์</u>

<u>ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยนเรศวร</u>

<u>คำชี้แจง</u>

1. แบบทดสอบพื้นฐานวิชาคณิตศาสตร์มี 3 ด้าน คือ
   - 1.1) หลักคณิตศาสตร์
   - 1.2) การวิเคราะห์
   - 1.3) พีชคณิต
2. ให้ผู้สมัครสอบทำข้อสอบทั้ง 3 ด้าน และ <u>ให้นำมาในวันสอบสัมภาษณ์</u>

ข้อสอบด้านหลักคณิตศาสตร์

1. Let A and B be sets. Prove that if $A \subseteq B$ then $A \cap B^C = \varnothing$.

2. Let a relation R on $\mathbb{Z}$ be defined by xRy if and only if $3 \mid (x-y)$

   Is R an equivalence relation ? Does R has an antisymmetric property ?

3. Let $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 1 - 3x^2\}$. Find the domain of R (Dom(R)) and the image of R (Im(R)) with proofs.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x^2 - 1$. Is f a bijection ?

5. Let $f : A \to B$ and $g : B \to C$. Prove that if f and g are one to one functions then $g \circ f : A \to C$ is also one to one.

ข้อสอบด้านการวิเคราะห์

1. Prove that $1^3+2^3+3^3+...+n^3=\frac{n^2(n+1)^2}{4}$ for all $n\in\mathbb{N}$.

2. Suppose that $\{a_n\},\{b_n\}$ and $\{c_n\}$ are the sequences of real numbers such that

   $\lim_{n\to\infty} a_n = a = \lim_{n\to\infty} c_n$ and $a_n \le b_n \le c_n$ for all $n\in\mathbb{N}$. Prove that $\lim_{n\to\infty} b_n = a$.

3. Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences. Prove that

   $\limsup_{n\to\infty}(x_n+y_n) \le \limsup_{n\to\infty} x_n + \limsup_{n\to\infty} y_n$.

4. Suppose that $S\subseteq\mathbb{R}$ and $f:S\to\mathbb{R}$ is a function with $c\in S.$ Prove that f is continuous at c if and only if for every sequence $\{x_n\}$ where $\{x_n\}\in S$ and $x_n\to c$, the sequence $\{f(x_n)\}$ converges to $\{f(c)\}$.

5. Let $I\subseteq\mathbb{R}$. Prove that if $f:I\to\mathbb{R}$ is differentiable at a point $c\in I$, then f is continuous at c.

ข้อสอบด้านพีชคณิต

1. Let H and K be subgroups of a group G. Prove that HK is a subgroup of G if and only if $HK = KH$.

2. Let $\mathrm{SL}(2,\mathbb{R}) := \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in \mathbb{R} \text{ and } \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 1 \right\}$, where $\mathbb{R}$ is a set of real numbers. Show that $\mathrm{SL}(2,\mathbb{R})$ is a group under multiplication of matrices.

3. Prove that every group is isomorphic to a subgroup of a permutation group.

4. Let R be a ring and let $\{S_i \mid i \in \mathrm{I}\}$ be a family of subrings of R. Prove that $\bigcap_{i \in \mathrm{I}} S_i$ is a subring of R.

5. Let $(R,+,\cdot)$ is a ring and I be an ideal of R.

   Let $R/\mathrm{I} = \{ a + \mathrm{I} \mid a \in R \}$. Define operations + and . on R/I by

   $$(a+\mathrm{I})+(b+\mathrm{I}) = (a+b)+\mathrm{I}$$
   $$(a+\mathrm{I})\cdot(b+\mathrm{I}) = (ab)+\mathrm{I} \quad ,\forall a,b \in R$$

   Prove that $(R,+,\cdot)$ is a ring.